CONCERNING INVERSIBLE FIBRATIONS, GLOBALIZATION AND TRIVIALITY OF MAPS

NIGAR TUNCER OZARSLAN

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Question *Let* $p: E \to B$ *be a locally trivial mapping and suppose that B is contractible. Does it follow that p is trivial?*

Definition The mapping $p: Y \to B$ is called *trivial* if there exists a homeomorphism $h: Y \to BxF$ for some space F such that $\pi_1(h) = p$

Corollary Assume that the space B is paracompact, Hausdorff and locally contractible in the large. Then p is an inversible fibration if and only if p is locally trivial.

E. Dyer and S.Eilenberg, *Globalizing fibrations with schedules*, Fund. Math. 130 (1988)

Definition Let $p: E \to B$ be a map and $\mathrm{F} = (E, p, B)$ be a space over B and $\Omega_p = \{(e, \omega) \in ExB^I | p(e) = \omega(0)\}$.

Let $\tilde{p}: E^I \to \Omega_p$ such that $\tilde{p}(\tau) = (\tau(0), p\tau)$ .F is called *Hurewicz fibration* if and only if there exists a map $\lambda: \Omega_p \to E^I$ such that $\tilde{p}\lambda$ is the identity map $\Omega_p \to \Omega_p$

Definition Let T denote the set of all real numbers $t \geq 0$ The path space PB of a topological space B is the subspace of $T \times B^T$ defined by

$$PB = \{(l, u) | u(t) = u\,(l), l \leq t\}$$

Let $p: E \to B$ be a space over B. The space $\widetilde{\Omega_p}$ is the subspace of $E \times PB$ of all pairs $(e, (l, u))$ such that $p(e) = u(0)$.

A mapping p is a *D&E fibration* if and only if there exists a continuous function (*called action*)

$$\star: \widetilde{\Omega_p} \to E$$

To be written $e \star (l, u)$, which satisfies the conditions

$p(e \star (l, u) = u(l)$ and $e \star \left(0, \widehat{p(e)}\right) = e$, where

$$\widehat{p(e)}(t) = \begin{cases} p(e) & , t = 0 \\ \widehat{p(e)}(0) & , t > 0 \end{cases}$$

A mapping is called *inversible fibration* if and only if there exists two reciprocal actions $\star_1, \star_2 : \widetilde{\Omega_p} \to E$ satisfying

$$\big(e \star_1 (l,u)\big) \star_2 -(l,u) = e = \big(e \star_2 (l,u)\big) \star_1 -(l,u)$$

for all $\big(e,(l,u)\big) \in \widetilde{\Omega_p}$. Where the inverse path is defined as $-(l,u) = (l,\bar{u})$,

$$\bar{u}(t) = \begin{cases} u(l-t), & 0 \le t \le l \\ u(l) & , l \le t \end{cases}$$

Regularity, inversibility, D&E fibrations, Hurewicz fibration

Definition A lifting function is called *regular* if it lifts constant paths to constant paths.

Definition An action is called *regular action* if and only if it satisfies the condition $e \star (l, \widehat{p(e)}) = e$ for all $l \in T$ where $\widehat{p(e)}(t) = p(e)$.

Definition A map is an *inversible Hurewicz fibration* if and only if there exists two lifting functions

$$\lambda_1, \lambda_2 : \Omega_p \to E^I \quad \text{satisfying}$$

$$\lambda_1(\lambda_2(e,\omega)(1), \varpi)(1) = \lambda_2(\lambda_1(e,\omega)(1), \varpi)(1) = e$$

for every $(e, \omega) \in \Omega_p$.

Definition A space over B is called *regularly inversible Hurewicz fibration* if and only if it is an inversible Hurewicz fibration with reciprocal regular lifting functions.

Theorem A space over B is an inversible D&E fibration then it is inversible Hurewicz fibration.

Theorem A space over B is a regular D&E fibration if and only if it is a regular Hurewicz fibration.

Theorem A space is regularly inversible D&E fibration if and only if it is regularly inversible fibration.

Theorem If B is paracompact and contractible and f is a space over B then the followings are equivalent;

(1) F is locally trivial,

(2) F is an inversible D&E fibration,

(3) F is an inversible Hurewicz fibration,

(4) f is trivial.

THANK YOU